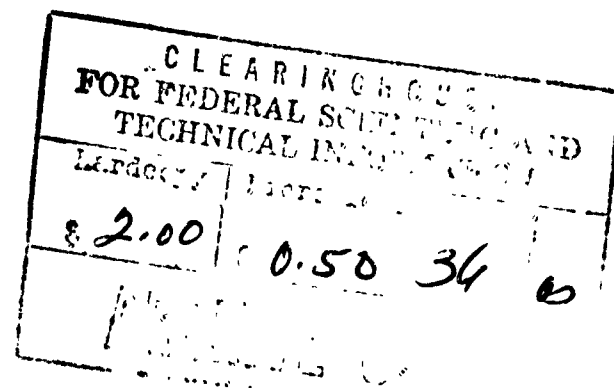# BOLT BERANEK AND NEWMAN INC

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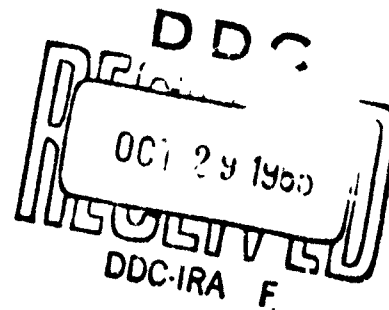Job.No. 11139

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THEORY OF BOUNDARY LAYER NOISE

Contract No. Nonr-4285(00)

13 July 1965

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Contract Research Administrator
Hydromechanics Laboratory
David Taylor Model Basin
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THEORY OF BOUNDARY LAYER NOISE

by

J. E. Ffowcs Williams

13 July 1965

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I. INTRODUCTION

During the present contract work of a fundamental nature has been
conducted into several aspects of the theory of boundary layer
induced sound and vibration. Each of the major subject areas has
proved to provide tractable theoretical problems which have led
to technical reports. Accordingly, those topics will only be
discussed very briefly and the abstract of the technical report
included in the text. The final technical report to come out
of the effort has not yet been submitted for publication because
of lack of time. That report is reproduced here in its entirety
as Section VI of this final report. Permission is sought to
have this report submitted for journal publication. In summariz-
ing the work accomplished during the contract period we must
inevitably recapitulate on some of the earlier progress reports
but the latter stages of the work have not previously been re-
ported to the David Taylor Model Basin.

The first stage of the research program was a study of the sound
radiated by turbulent flow formed on infinitely large homogeneous
plane surfaces whose response to the surface pressure could be
varied by changing the structural properties. Several interest-
ing points have emerged from that analysis. Quite contrary to
our initial expectation, there are no simple sources or surface
dipoles associated with the problem, within the framework of
the acoustic analogy. We know this to be the case when the sur-
face is rigid, for the surface then acts as a passive reflector.
But more surprising is the result that when the surface is per-
fectly limp and appears as a pressure relief condition, we again
get perfect reflection of the volume quadrupoles. But this time
the images have precisely the opposite strength, a result familiar
in purely acoustical situations, but rather surprising in this
instance where an intense hydrodynamic field excites the sound.

At intermediate surface conditions the general principle is still
valid that no simpler and more efficient sources than those present
in the turbulence are required to represent the total radiation
field. In fact the surface acts as a passive reflector but the
reflection coefficient is complex and varies with the angle of
incidence of the reflected wave. The reflection coefficient is
precisely that relevant to reflection of plane acoustic waves so
that the boundary does not appear as a real source of radiation.
This work is written up in a recently published report (Journal of
Fluid Mechanics, June 1965) and the summary is included in the text
of this report.

The second item that has been studied under this current con-
tract is the nature of the sound radiated by the boundary stresses
if these stresses are computed on the basis of the incompressible
flow equations. A large, if not a dominant, fraction of the wall
pressure arises from the interaction of turbulent eddies with the
large mean shear of the boundary layer flow. The sound radiated
by this type of pressure field has been studied in detail. The
wall pressure appears to be equivalent to a distribution of
lateral quadrupoles of strength proportional to the mean shear.
One cannot draw defininte conclusions from this work at present,
but it appears possible that the lateral quadrupoles Lighthill
showed to dominate the turbulent field whenever the mean velocity
gradients were high, whose strength increased in direct proportion
to the mean shear, is being opposed by the image-like lateral
quadrupoles equivalent to the mean shear induced surface pressure
field. If this is the situation, it may be that the reflection
criterion is already established even though the surface pressure,
a near field phenomenon in this instance, is an extremely local
one.

This is only one of the points described in a general survey
Monograph on the sound generated by turbulent boundary layers.
That monograph is published as AGARDograph 90 and its summary
is included in this present text. It is pertinent to say that
reviewers' comments on this work have, to date, been very compli-
mentary describing it as a work likely to remain a standard
reference for many years to come.

As part of the effort aimed at clarifying the role of incompres-
sible flow arguments in the theory of acoustic radiation from
turbulent boundary layers, an extensive study of how compres-
sibility effects influenced the wall pressure was undertaken.
The main conclusion was that it was precisely those usually in-
significant features of the pressure field affected by compres-
sibility that determined the radiation properties of the wall
pressure field. Consequently it is essential to consider the
influence of finite Mach number effects when considering radia-
tion from turbulent flow or plane surfaces. The outcome of the
work was reported on two occasions and appears published in
papers that are similar but differ in detail, the latter publi-
cation being more rigorous. The first report appears in the
published proceedings of the second international conference on
acoustical fatigue and the abstract of the paper is included in
the text. The second and more carefully considered report will
appear in the July 1965 issue of the Journal of Fluid Mechanics
and again the abstract of that work is included.

The final significant study to be undertaken under this contract
was to study the effect of idealized supports on the sound
radiated from an otherwise uniform homogeneous structure. The
aim here was to maintain the exact approach made possible by the

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choice of a suitably simple model structure to study how the support
extracted energy from the turbulence that enabled the surface terms
to represent a fundamentally more efficient radiator then was
present in the unsupported surface. The mechanism is evidently a
wave scattering process whereby components of the surface response
that do not normally contribute to the radiation field, are
coupled to the radiation by the point scatterer. This process
destroys any significant correlation between the volume and sur-
face sources so that no subtle cancellation effect that might limit
the usefulness of approximate calculations need be expected. It
was the neglect of precisely such a cancellation that led to the
erroneous 'dipole' computations of boundary layer noise that were,
until quite recently, prevalent in the literature. A report on
this work is now ready for submission to a technical journal.
Section VI of this final report is that report.

Finally, this report is concluded by brief reference to other
topics that have been considered under this programme but which
have not led to significant new conclusions. That is followed
by a passing comment on some of the questions that remain
outstanding in this field.

II. MECHANISM OF NOISE GENERATION IN A TURBULENT BOUNDARY LAYER

[Preface to AGARDograph 90]

The subject matter treated here is a comparatively new and
rapidly expanding one. The bulk of the published works on
boundary layer noise appeared in the last four or five years.
Under these circumstances the writer of a monograph faces the
risk of becoming rapidly out-dated. Recognizing this situation,
the authors did not try to cover all the available literature
on boundary layer noise production in detail - except by giving
an extensive list of references - but rather they attempted to
pick out those works that in their opinion were most instru-
mental in generating new ideas and approaches to this difficult
problem. The arrangement of the chapters reflects this point
of view.

Chapters 1, 2 and 3 are introductory ones: the first chapter
briefly states the problem and gives its historical development;
the second reviews some basic notions of classical acoustics
with special emphasis on the sound field produced by elementary
sound sources, while the third one contains the generalized wave
equation governing the pressure field radiated by a nonuniform,
nonstationary flow. The subsequent chapters describe then the
methods proposed by various authors for finding a solution to
the radiation problem.

Undoubtedly, the theory that exerted the most profound influence
on the subject is Lighthill's acoustic analogy. Although the
theory has been described in a number of papers by Lighthill
himself and by several subsequent workers, it has often been
misinterpreted and misused. Chapter 4 attempts to restate again

the analogy, summarizing all the assumptions involved and
endeavors to point out the advantages and disadvantages of the
approach. In the same chapter Ribner's interpretation of the
source term in the analogy is touched upon, but only briefly.

Chapter 5 describes the application of the analogy to the
boundary layer problem. The importance of the size of the
boundary surface compared to the radiation wavelength is
emphasized.

Chapter 6 introduces a new approach proposed by Phillips to
treat the radiation problem for high convection velocities.
Some comparisons are made between his theory and the conse-
quences Ffowcs Williams drew from Lighthill's analogy applied
to higher convection speeds.

A separate section, Chapter 7, is devoted to a third method
of attack to study aerodynamic noise first proposed in an
unpublished work by Liepmann. This approach, although virtu-
ally unexplored, offers a possibility of relating the radiated
noise to flow parameters familiar in the study of incompressible
flows. It is this aspect, with its promise of straight-forward
experiments, that led the authors to devote to it a complete
chapter, although much of the chapter is of a general
illustrative nature.

The last chapter deals with the experimental investigations
concerning boundary layer noise. It is to be noted that the
considerable wealth of information on fluctuating velocity field
within a boundary layer and on the pressure fluctuations over

the solid surface adjacent to the layer obtained at subsonic
speeds have not been included. There are several indications,
both theoretical and experimental, that such subsonic fields
radiate very small noise indeed and have no practical signifi-
cance. For this reason the chapter concentrates primarily on
the supersonic boundary layer problem.

The monograph is designed for those who are familiar with
classical acoustics and fluid mechanics in addition to some
basic notions of turbulence: the concepts of correlation and
spectrum functions. Wherever a more detailed treatment of a
particular question exists in the literature, an attempt was
made to give adequate references. The bibliography does not
include all the papers on aerodynamic noise but only those
restricted to boundary layers. In this sense it is hopefully
fairly complete.

III.   VIBRATION INDUCED BY BOUNDARY LAYER TURBULENCE

[Abstract of Paper presented at the Second
International Conference on Acoustical Fatigue]

The vibration of a large homogeneous panel excited by a uniform
turbulent boundary layer is described in detail. The simplicity
of this situation allows one to appreciate the principal features
of the boundary layer flow that bring about structural vibration.
The large-surface result is then used as a guide to the study of
smaller panels. It is shown how important features of the flow
assume a different relative significance in studying the response
of inhomogeneous structures. A general classification of the
important vibration regimes is outlined and the experimental
turbulence data useful in determining the vibration levels in
these regimes are summarized.

IV.   SURFACE-PRESSURE FLUCTUATIONS INDUCED BY BOUNDARY-LAYER
FLOW AT FINITE MACH NUMBER

A theory describing boundary-layer surface-pressure fluctuations
on a rigid surface is presented in a form that illustrates the
main effect of compressibility. The most significant effect is
that the correlation area is proportional to the square of mean-
flow Mach number so it does not vanish in flow of finite com-
pressibility. Modifications of the wave-number and frequency
spectra by this effect are described, and the results applied
to the computation of large plate response. That computation
incorporates the effect of fluid loading, which enters the
response equations as a dissipative term for components at super-
sonic phase velocity but merely as an added loading for
subsonic components.

V. SOUND RADIATION FROM TURBULENT BOUNDARY LAYERS FORMED
ON COMPLIANT SURFACES

The paper considers the effect of turbulence-induced surface
response on the sound radiated by a turbulent boundary layer.
The analysis is confined to an infinite plane homogeneous sur-
face and the conclusions may not be a good indication of the
behavior of more realistic structures. The main result of the
analysis is that no fundamentally more efficient source of
sound is introduced by the surface motion. The radiation
remains quadrupole in character. The surface merely accounts
for a reflection of the turbulence-generated sound, with the
reflection coefficient being identical to that of plane acoustic
waves. Dissipation in the surface reduces the magnitude of the
image system. A brief discussion of the effect on the particu-
lar quadrupoles to be found in a turbulent boundary layer
concludes the paper. There it is argued that the radiation
will probably be increased by surface motion, but not by an
order of magnitude.

VI. THE INFLUENCE OF SIMPLE SUPPORTS ON THE RADIATION FROM TURBULENT FLOW NEAR A PLANE COMPLIANT SURFACE

ABSTRACT

The paper deals with an extension of previous work on the radiation properties of turbulent flow formed on compliant surfaces. The effect of simple supports is shown to be acoustically equivalent to an extended dipole system of strength equal to the support stress. The dipole radiation is reduced by a transmission factor below that radiated into an uniform environment. A particular example is worked out in detail. That example deals with the case of a single point support on an otherwise homogeneous surface excited by boundary layer turbulence.

1. The Dipole Equivalent of Simple Supports.

The influence of surface vibration on the radiation from turbulent flow near a homogeneous plane surface has recently been treated by Ffowcs Williams (1965). There, it was shown that surface response did not introduce sources of high efficiency, and that any surface effect could be accounted for by a straight-forward reflection coefficient for plane acoustic waves. The influence of simple supports can be treated in a similar way. Again, nonlinear terms in surface response are neglected, as are the viscous terms. The equations that describe the radiation field are those given by Powell (1961). There are two complimentary equations, one for a real flow with turbulence stress tensor T_{ij} distributed over the volume v_{+}, and one for a hypothetical image flow with a specular reflection of the turbulence in the volume v_{-}.

$$p(\underset{\sim}{x},t) = T_+ + p_r + p_v$$

$$0 = T_- - p_r + p_v$$

$$T_\pm = \frac{1}{4\pi} \frac{\partial^2}{\partial x_i \partial x_j} \int_{v\pm} [T_{ij}] \frac{dy}{r} \; ;$$

$$p_r = \frac{1}{4\pi} \frac{\partial}{\partial x_n} \int_s [p] \frac{dy}{r} \; ;$$

$$p_v = - \frac{1}{4\pi} \int_s \frac{1}{\rho} \frac{\partial[v_n]}{\partial t} \frac{dy}{r}$$

(1.1)

The brackets, [ ], indicate that the integrals should be evaluated at retarded time, $(t-r/a_o)$, r being the distance separating the source point $\underset{\sim}{y}$ from the observer at $\underset{\sim}{x}$, and a_o the speed of sound. n is the outward normal from the real flow through the plane bounding surface s. $p(\underset{\sim}{x},t)$ is the pressure radiated to the point $(\underset{\sim}{x},t)$ and comprises four distinct source terms. The quadrupoles in the real flow induce a pressure T_+ while those in the image flow induce a pressure T_-. Should the surface be rigid, surface sources would account for a pressure p_r, while a pressure p_v would be radiated by surface terms if the surface were perfectly limp and could support no stress. More general surfaces radiate sound in a way that is determined by Eq. 1.1 so that the problem is reduced to one of relating the two pressures p_r and p_v through some knowledge of the surface response.

We now suppose that the previously considered homogeneous plane surface is supported by an inhomogeneous stress system induced by a distribution of simple supports. These stresses will be denoted by q, a positive q implying a force acting on unit area of the surface in the direction, -n. The stresses are related to the response velocity, v_n, through the response equation,

$$p - q = F(v_n) \quad , \tag{1.2}$$

F being a collection of differential or integral operators representing a linear integro-differential equation with constant coefficients. The surface pressure is now eliminated from 1.1 by use of the response equation.

$$p_r = \frac{1}{4\pi} \frac{\partial}{\partial x_n} \int_s [p] \frac{\widetilde{dy}}{r} = \frac{1}{4\pi} \frac{\partial}{\partial x_n} \int_s [q] \frac{\widetilde{dy}}{r} + \frac{1}{4\pi} \frac{\partial}{\partial x_n} \int_s [F(v_n)] \frac{\widetilde{dy}}{r} \quad . \tag{1.3}$$

This relation reduces the current problem, by analogy, to that of the unsupported homogeneous surface. This becomes clear when we regard the support stress, q, as the strength of a distribution of externally applied acoustic dipoles whose total strength we do not expect to be generally zero. These external dipoles are essentially different from the remaining surface terms, representing real sources of radiation and not, in general, accounting for a reflection property which is the sole role of the other terms. This point will become clear in what follows but we anticipate it by combining the total effect of the applied stress fields into one term. That term represents the sum of the pressure induced by the turbulent flow and that induced by the support dipoles, a value we denote by S. As before, we have a field due to the real and image source systems.

$$S_+ = T_+ + \frac{1}{4\pi} \frac{\partial}{\partial x_n} \int_s [q] \frac{dy}{\underset{\sim}{r}}$$

$$(1.4)$$

$$S_- = T_- - \frac{1}{4\pi} \frac{\partial}{\partial x_n} \int_s [q] \frac{dy}{\underset{\sim}{r}} \quad .$$

S_- is the pressure induced by the specular reflection of the real
source system that generates the pressure S_+, the reflection of the
dipole term merely requiring a change of sign.

Equation 1.1, 1.3 and 1.4 can be combined in a form that makes
clear the analogy with the earlier problem.

$$p(\underset{\sim}{x},t) = S_+ + \frac{1}{4\pi} \frac{\partial}{\partial x_n} \int_s [F(v_n)] \frac{dy}{\underset{\sim}{r}} - \frac{1}{4\pi} \int_s \bar{\rho} \frac{\partial[v_n]}{\partial t} \frac{dy}{\underset{\sim}{r}}$$

$$(1.5)$$

$$0 = S_- - \frac{1}{4\pi} \frac{\partial}{\partial x_n} \int_s [F(v_n)] \frac{dy}{\underset{\sim}{r}} - \frac{1}{4\pi} \int_s \bar{\rho} \frac{\partial[v_n]}{\partial t} \frac{dy}{\underset{\sim}{r}} \quad .$$

This system of equations is precisely that treated by Ffowcs
Williams (1965) in considering the problem of turbulent flow
formed on an unsupported homogeneous surface. In fact the only
change induced by the supports is that the turbulent sources are
reinforced by surface dipoles so that S replaces T. Conclusions
can therefore be based directly on that analysis. The most
important point is that the surface integrals account for simple
reflection of the source system S_+. However the reflection co-
efficient changes with direction of radiation and frequency. The
analysis is particularly simple for the distant radiation field

where, $p^*(\underset{\sim}{x},\omega)$, the component of radiated pressure at frequency ω is given by the sum of direct and reflected fields.

$$p^*(\underset{\sim}{x},\omega) = S^*_+ + R\ S^*_- \quad . \tag{1.6}$$

S^*_+ and S^*_- are the component of sound pressure radiated by the real and image source systems at frequency ω, and R is the reflection coefficient for plane acoustic waves at that frequency.

The pressures S^*_+ and S^*_- consist of a superposition of the fields induced by quadrupoles acoustically equivalent to the turbulent flow and dipoles whose strength density equals the supporting stresses. Equation 1.6 has an interesting special case when there is negligible turbulence so that both T_+ and T_- are zero. Then S^*_+ is entirely due to an externally applied stress and S^*_- is its exact opposite, being the field of an image dipole. The radiated pressure is then given by,

$$p^*(\underset{\sim}{x},\omega) = (1-R)\ S^*_+ \quad . \tag{1.7}$$

(1-R) is familiar as the transmission coefficient for waves passing from the fluid into a region with impedance equal to the normal impedance of the surface. This result, that the radiation from an externally excited surface is equal to that induced by dipoles of strength density equal to the applied stresses multiplied by the transmission coefficient for plane acoustic waves, seems an obvious one but does not appear to be readily available in the literature.

It is clear that if the supporting stresses were uniformly dis-
tributed over the plane their effect could be accounted for by a
modified surface response equation. The previous conclusion that
the radiation would be purely quadrupole would then be valid. It
seems that it is essentially the inhomogeneous nature of the sup-
ports that induces the dipole component. If the plane surface
were composed of several regions of locally homogeneous material,
but material that differed from region to region, one could con-
clude from the foregoing analysis that within individual regions
the effect of surface motion would be accounted for by the local
reflection coefficient and that the radiation would be quadrupole.
However at the interfaces there would be discontinuities in the
response equation that would account for dipole terms which must
be more efficient radiators of sound. The situation is completely
analogous to that treated by Maidanik (1962) who showed how most
of the sound radiated from a large finite plate appeared to
emanate from the periphery of the plate.

The total dipole strength is the net applied force and the most
effective radiation results from the force being concentrated on
to an area of typical dimension small in comparison with an acous-
tic wavelength. The concentrated point support is then an
important example in establishing an upper limit on the strength
of the radiation field induced by a known supporting force. When
the excitation originates in boundary layer turbulence the sup-
porting force is not known a priori, unless by experiment. The
example then offers no specific upper limit on the radiation but can
still serve as a useful result for developing more pertinent
models of real system. That example is considered below.

2.   Point Supported Surface Under a Turbulent Boundary Layer

The simplest of the inhomogeneous support system is that in which the stress distribution q is concentrated at one point. Let that point be the origin of co-ordinates and let the applied force have a value Q.

$$q(\underset{\sim}{y}, t) = Q(t)\ \delta(\underset{\sim}{y}) \quad . \tag{2.1}$$

This force induces a dipole radiation according to Eqs. 1.4 and 1.5, where the real and image fields become:

$$S_+ = T_+ + \frac{1}{4\pi} \frac{\partial}{\partial x_n}\left[\frac{Q}{r}\right]$$

$$S_- = T_- - \frac{1}{4\pi} \frac{\partial}{\partial x_n}\left[\frac{Q}{r}\right] \tag{2.2}$$

In the distant radiation field, the differentiation with respect to x_n applies only to the retarded time so that the dipole term may be rewritten as,

$$\frac{1}{4\pi r} \frac{\sin\theta}{a_o} \frac{\partial Q}{\partial t} \quad . \tag{2.3}$$

$\sin\theta$ is written for $-\frac{\partial r}{\partial x_n}$, θ being the radiation angle measured from the surface. The spectral decomposition is achieved through Fourier transformation. We shall denote transformed quantities by an asterisk, p* being the component of p at frequency ω, T_+^* the component of T, etc.

$$p(\underset{\sim}{x}, t) = \int p^*(\underset{\sim}{x}, \omega) e^{i\omega t}\ d\omega \quad . \tag{2.4}$$

The particular case of Eq. 1.6 is then,

$$p*(\underset{\sim}{x},\omega) = T_+^* + R\ T_-^* + \{1-R\}\frac{i\omega\ \sin\theta}{4\pi\ a_o r}\ Q* \quad . \qquad (2.5)$$

It is easy to verify that the dipole term is precisely that worked out by (Maidanik and Kerwin, Jr.) to be the radiation from a point driven plate, but its interpretation as the sum of direct and reflected dipole fields seems to be new. A point of considerable interest is the question of how large is the dipole field in comparison with the sound radiated by the surrounding surface? To progress with that issue, the value of the externally applied force, Q, must first be found.

Suppose the support to have some impedance z_q, so that $Q*$ is related to the velocity at the support point v_{nq}^*, through the relation:

$$Q* = z_q\ v_{nq}^* \quad . \qquad (2.6)$$

The applied force has modified the velocity at its point of application, from a value v_{no}, which would have occurred in an unsupported structure, to its current value v_{nq}. The force is related to this velocity change through z_p, the point impedance of the structure, an impedance that includes any influence of fluid loading.

$$Q* = -\ z_p(v_{nq}^* - v_{no}^*) \quad . \qquad (2.7)$$

Combining this relation with Eqs. 2.5 and 2.6 we obtain expressions for the applied force and radiated sound in terms of the velocity response of an unsupported structure.

$$Q^* = \frac{z_p}{\left\{1 + \frac{z_p}{z_q}\right\}} v^*_{no} \tag{2.8}$$

$$p^*(\underset{\sim}{x},\omega) = T^*_+ + R\, T^*_- + (1-R)\, \frac{i\omega\, \sin\theta}{4\pi\, a_o\, r}\, \frac{z_p\, z_q}{(z_p + z_q)}\, v^*_{no} \quad . \tag{2.9}$$

The influence of the support can now be interpreted as a wave scattering process. Sound radiation from an unsupported surface occurs only from those spectral components that match both frequency and wave number of the distant acoustic wave. But in this instance, components of response velocity at all wave numbers contribute to the term v^*_{no}, so that the supported structure acts like a sounding board. That is, energy is converted from a reactive to a radiative regime by a wave scattering process. This feature destroys any significant correlation between the dipole and quadrupole terms making the mean square radiation the sum of the individual mean square values. The power spectral density of the radiated pressure field $P^*(\underset{\sim}{x},\omega)$, can then be expressed as the sum of that due to the combination of real and image quadrupoles T^*, and that due to the dipole which is proportional to the power spectral density of the response velocity in an unsupported panel, V^*_o.

$$P^*(\underset{\sim}{x},\omega) = T^* + |1-R|^2\, \frac{\omega^2\, \sin^2\theta}{16\pi^2\, a_o^2\, r^2}\, \left|\frac{z_p\, z_q}{z_p + z_q}\right|^2\, V^*_o \quad . \tag{2.10}$$

V^*_o is simply related to the three dimensional Fourier spectrum of the pressure field acting on a rigid surface through the surface

and wave impedances, z and z_w respectively. The rigid-surface pressure field at wave vector $\underset{\sim}{k}$ and frequency ω, $p_{rs}^*(\underset{\sim}{k},\omega)$, must balance both the structural response force, $z\, v_n^*(\underset{\sim}{k},\omega)$, and the force induced by fluid motion, $z_w\, v_n^*(\underset{\sim}{k},\omega)$. Therefore:

$$p_{rs}^*(\underset{\sim}{k},\omega) = (z + z_w)\, v_n^*(\underset{\sim}{k},\omega) \quad , \tag{2.11}$$

where $v_n^*(\underset{\sim}{k},\omega)$ is the three dimensional Fourier transfrom of the surface velocity. The three dimensional spectral functions, which we denote $P_{rs}^*(\underset{\sim}{k},\omega)$ and $V_n^*(\underset{\sim}{k},\omega)$, are formed from the product of this equation with its complex conjugate

$$P_{rs}^*(\underset{\sim}{k},\omega) = |z + z_w|^2\, V_n^*(\underset{\sim}{k},\omega) \quad . \tag{2.12}$$

The frequency spectrum of the surface response velocity is simply the integral of the three dimensional spectrum over all wave number space so that $V_o^*(\omega)$ is given by a straightforward integral.

$$V_o^*(\omega) = \int \frac{P_{rs}^*(\underset{\sim}{k},\omega)}{|z + z_w|^2}\, d\underset{\sim}{k} \quad . \tag{2.13}$$

For a homogeneous structure the impedance function depends only on the magnitude of the wave vector so that a change of coordinates to a polar system is suggested. We let $\underset{\sim}{k}$ be defined by (k,ϕ) and rewrite $d\underset{\sim}{k}$ as $kdk\,d\phi$.

$$V_o^*(\omega) = \int_o^\infty \frac{k\, dk}{|z + z_w|^2} \int_o^{2\pi} P_{rs}^*(\underset{\sim}{k}(k,\phi),\omega)d\phi \quad . \tag{2.14}$$

In a highly resonant structure, the integral over wave number can be approximated in a way that considerably simplifies the analysis. The approximation rests on the assumption that both the pressure spectrum and the real part of the impedance $(z + z_w)$, $(z_R + z_w)$, $(z_w$ being necessarily real for radiating waves) remains fairly constant over the effective bandwidth of the 'resonance peak,' a peak assumed to occur at wave number k_p. In that event, the integration over wave number is straightforward.

$$V_0^*(\omega) = \frac{2\pi}{[z_R + z_w]z_p} \int_0^{2\pi} P_{rs}^*(\underset{\sim}{k}(k_p,\phi),\omega)\, d\phi \quad . \tag{2.15}$$

Most turbulent flows of practical interest display convective features that are apparent in the spectrum function as a tendency for the energy to be concentrated at the eddy passage frequency. This property can be important in the response problem and is best dealt with by re-expressing the pressure spectrum in terms of that measured by an observer in uniform motion with the most coherent eddy structure. That spectrum we will denote by $P_m^*(\underset{\sim}{k},\omega)$.

$$P_{rs}^*(\underset{\sim}{k},\omega) = P_m^*(\underset{\sim}{k},\omega - \underset{\sim}{k}\cdot\underset{\sim}{U}_c) \quad . \tag{2.16}$$

U_c is the convection velocity which we normalize with respect to the free surface wave speed, c_p. $|U_c| = c_p M$. Then, by setting the origin of ϕ coincident with the direction of convection and noting that $\omega = k_p c_p$, we can rewrite Eq. 2.15 in a form that displays the convective effects more clearly.

$$V_0^*(\omega) = \frac{2\pi}{[z_R + z_w]z_p} \int_0^{2\pi} P_m^*(\underset{\sim}{k}(k_p,\phi),\, \omega[1 - M\cos\phi])d\phi.$$

$$\tag{2.17}$$

At low flow velocities, particularly in underwater applications, interest centers on situations where the number M is negligible. Then it is convenient to assume that space and time variables are separable in the moving reference frame so that:

$$P_m^*(\underset{\sim}{k}(k_p, \phi)), \ \omega(1 - M \cos\phi) = p_h^2 \ P_{\underset{\sim}{k}}^*(\underset{\sim}{k}) \ P_\omega^*(\omega) \quad . \tag{2.18}$$

p_h is written for the r.m.s. pressure level active on a rigid boundary, $P_{\underset{\sim}{k}}^*$ is the wave number spectrum and P_ω^* is the moving axis frequency spectrum. Both these spectral functions are normalized to integrate to unity.

Before going on to evaluate the integral at low values of M, it is worth pointing out the equivalence that exists between this theory of vibration induced by convected pressure fields and that of aerodynamic sound generation by convected turbulence. (Lighthill 1962, Ffowcs Williams 1963). In the aerodynamic case, M is the Mach number of eddy convection, and radiation frequencies differ from those of the source by the Doppler factor $(1-M \cos\phi)$. It is apparent from Eq. 2.17 that this feature is also a property of the vibration problem and that we might expect an analogue of the Mach wave radiation at values of M in excess of unity. This is evident from the alignment that occurs whenever $(1-M \cos\phi)$ approaches zero of the dominant spectral component in the surface pressure field with the response frequencies. The spectrum P_ω^* being chosen so that its maximum occurs at zero frequency. Then, by analogy with the aerodynamic problem, only the uniformly convected components induce response. Consequently the response would be expected to be relatively intense for those waves radiating at the "Mach angle" on account of the strong tendency to uniform convection of

many pressure fields. The situation is illustrated quite effec-
tively by assuming the moving axis frequency spectrum to be a unit
delta function. Then Eq. 2.17 can be evaluated to give the
response velocity spectrum typical of structural excitation by
high speed flows where the convection velocity exceeds the phase
speed of free waves.

$$V_j(\omega)_{(1-M\cos\phi)=0} = \frac{2\pi \, p_h^2 \, P_k^*(k_p, \phi = \cos^{-1} M^{-1})}{\sqrt{M^2-1} \, \{z_R+z_w\} \, z_p} \quad . \tag{2.19}$$

In underwater problems the low speed situation is more relevant,
where, from Eqs. 2.17 and 2.18,

$$V_o^*(\omega) = \frac{2\pi \, p_h^2 \, P_\omega^*(\omega)}{\{z_R+z_w\}z_p} \int_0^{2\pi} P_k^*(k(k_p,\phi)) \, d\phi \quad . \tag{2.20}$$

The integral over ϕ is the correlation area $A(k_p)$ used by Ffowcs
Williams and Lyon (1963) so that it is a previously estimated
property of boundary layer flows. The values for $A(k_p)$ are
illustrated in Fig. 1.

$$2\pi \int_0^{2\pi} P_k^*(k(k_p,\phi)) d\phi = A(k_p) \quad . \tag{2.21}$$

The response spectrum can now be given explicitly in terms of known
features of the pressure field that acts on a rigid surface. The
spectrum has the value derived for the flat plate by Ffowcs
Williams and Lyon (1963) when z_w and z_p are appropriately chosen.

The radiation from the simply supported surface can also be
estimated by inserting the response spectrum in Eq. 2.10.

$$V_0^*(\omega) = \frac{p_h^2 \, P_\omega^*(\omega) \, A(k_p)}{(z_R + z_w) \, z_p} \tag{2.22}$$

$$P^*(\underset{\sim}{x}, \omega) = T^* + |1-R|^2 \, \frac{\omega^2 \sin^2\theta}{16\pi^2 \, a_0^2 \, r^2} \, \frac{z_p}{(z_R + z_w)} \, \left|\frac{z_q}{z_q + z_p}\right|^2 \, p_h^2 \, P_\omega^*(\omega) \, A(k_p) \,. \tag{2.23}$$

Although this result could be used to evaluate the sound radiated
from various flows and support structures, the general form is not
very revealing of the important role played by surface inhomogene-
ities. It is clear that if the surface is supported on soft, or
resonant undamped mounts, so that z_q approaches zero, there will
be no additional radiation from the support. It is also clear
that the support plays a minor role in high impedance structures
where the reflection coefficient approaches unity. However, many
practical instances occur where the reflection coefficient is
close to zero, as in the case in sonar dome construction where
optimum sound transmission is essential. That situation is illus-
trated below by an example in which the support impedance z_q is
infinite, the surface is assumed to be loss-free, and the flow is
a fully developed turbulent boundary layer. The intensity of the
radiation from the turbulence is identical to that from the image
system which can be computed from a knowledge of the pressure
field on the boundary surface. This has been done in an approxi-
mate form by Ffowcs Williams and Lyon 1963 with the result:

$$T^* = \frac{p_h^2}{16\pi^2} \, \frac{\cos^2\theta \, \sin^2\theta}{} \, 880\pi \, \frac{\delta_1^4}{\lambda^4} \, P_\omega^*(\omega) \, \frac{A}{r^2} \,. \tag{2.24}$$

The notation is that already defined with the addition that δ_1 is the boundary layer displacement thickness, λ is the inverse acoustic wavenumber a_o/ω, and A is the area of the radiating surface. This estimate, when inserted in Eq. 2.23 yields an approximate expression for the total radiation from a turbulent boundary layer formed on a surface resting on a single rigid support.

$$P*(\underset{\sim}{x},\omega) = \frac{p_h^2}{16\pi^2} \frac{\delta_1^4}{r^2\lambda^2} \sin^2\theta \; P_\omega^*(\omega)\left\{880\pi \frac{A}{\lambda^2} \cos^2\theta + \frac{z_p}{\delta_1^2 \, z_w} \frac{A(k_p)}{\delta_1^2}\right\} \quad .$$

$$(2.25)$$

The leading term in the brackets is that due to the free surface and the other represents the support. When this expression is evaluated for a typical underwater situation one finds that the support induces an intensity at $\theta = 45°$ equivalent to that radiated by approximately 2 square metres of unsupported structure. This figure is worked out near the maximum value of $A(k_p)$ which occurs at a frequency of 3.5 kc for 0.25 inch steel plate and the boundary layer displacement thickness is taken as 0.1 inch.

A more general result showing the area of free surface to which the radiation from the support is equivalent at a particular radiation angle is obtained by equating the two terms within the brackets of Eq. 2.25. The value of $A(k_p)/\delta_1^2$ has been taken as 10, that being an estimate of its upper limiting value as shown in Fig. 1. Then it is seen that at an angle θ the radiation from a rigid support on a surface with high transmission factor cannot exceed that from an unsupported area equal to,

$$\frac{1}{88\pi \cos^2\theta} \frac{z_p}{z_w} \frac{\lambda^2}{\delta_1^2} \quad . \qquad\qquad (2.26)$$

ACKNOWLEDGMENTS

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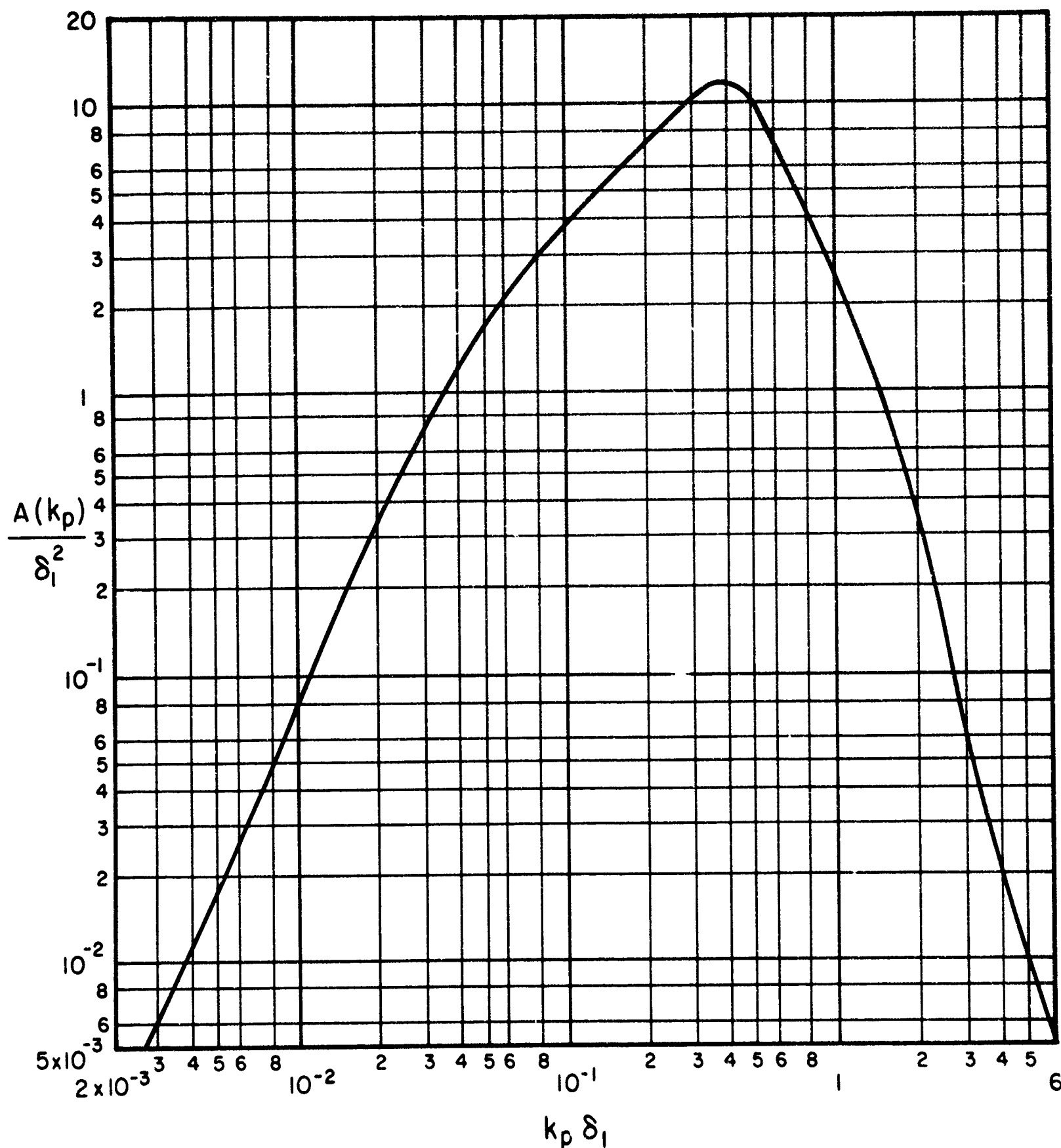   J. Acoust. Soc. Amer. 32, 8, 982.

FIG.1 FFOWCS WILLIAMS AND LYON'S (1963) ESTIMATE
OF THE EQUIVALENT CORRELATION AREA $A(k_p)$
(DEFINED BY EQ. 2.21 OF TEXT).

VII. CONCLUSION AND DISCUSSION

In the general area of sound radiation by turbulent flow interacting
with flexible surfaces it would seem helpful to distinguish between
two essentially different situations. The discussion arises in es-
tablishing whether or not the sources induced by surface terms
arise from hydrodynamic or compressible features of the turbulent
field. It is confusion on this issue that has been primarily
responsible for the erroneous estimates of radiation strength that
have appeared in the literature from time to time.

There seems to be no doubt at all that once the vibration field of
a structure is established, the radiation from the induced vibration
is accessible by what are now standard techniques. The influence
of compressibility is simply and properly accounted for in those
procedures. It is in the computation of the surface stresses and
response that the confusion arises. Two examples of this situation
are to be seen in recent survey articles on the subject. The first
is due to M. J. Lighthill in the Bakerian Lecture to the Royal
Society, where the radiation from a rigid surface was estimated to
be dipole in character and the dipole strength computed from
boundary layer experiments that were not intended to be accurate
in regimes where any effects of compressibility were significant.
It is now known that the situation treated by Lighthill is entirely
dependent in those usually small effects arising from fluid com-
pressibility so that his computation becomes meaningless. The
second example is due to Ffowcs Williams and Lyon in an AGARDograph
on aerodynamic noise edited by G. M. Lilley. There the radiation
from an infinite plane homogeneous structure was computed by
equating input power to the vibration field to the energy loss
by radiation. The input power was computed on the basis of the

equations of incompressible fluid motion and the result is quite
an erroneous overestimate by several orders of magnitude (at least
the square of Mach number) of the true radiation field. Again, it
is now known that the properties of the vibration field responsi-
ble for radiation are intimately connected with the sound field of
the turbulence and are rather unrelated to the hydrodynamic near
field. Both these points are evident from the papers whose
abstracts form sectionↄ IV and V of this report.

On the other hand where surface inhomogeneities occur there is
very little that is new to be gained from a more precise treatment
that consider effects of compressibility. This is evident from
section VI of this report where both the vibration level and the
radiation from the point support that are calculated are quite
uninfluenced by fluid compressibility. In fact the vibration
level is that previously calculated by Ffowcs Williams and Lyon
in the above mentioned AGARDograph, and the radiation from the
support is the familiar radiation from a point driven structure.

The essential feature determining whether or not compressibility
terms should be considered in computations of the surface stress
field is whether or not there is a unique wave matching between
the excitation and radiated field. If such a matching exists, as
is the case in the homogeneous surface, compressibility effects
must play a major role in establishing the significant components
of surface stress and response. On the other hand where there
is not a unique matching, and energy is scattered from one
spectral range to another, the relatively low intensity region
of the spectrum where compressibility effects are important can
play no major role in the radiation problem. The radiation
energy is simply acquired from the more intense spectral regions
by a scattering process.

Intermediate regimes are more difficult to deal with. The line
supported structure acts as a wave scatterer to waves travelling
in a direction normal to the support but cannot scatter waves
travelling parallel to the support. In a similar way the edge of
an otherwise uniform surface behaves as a wave scatterer to a
degree depending on direction. The calculation of the radiation
field by turbulent flow acting on such structures would conse-
quently require that the theory take into account the sound field
of the turbulence in addition to its hydrodynamic field. Of
course it could be argued that the radiation from structures that
do not possess significant scattering devices is essentially
quadrupole and must be insignificant in any practical underwater
application. There would seem no good reason to doubt this point
of view so that the emphasis should be placed on the reduction of
inhomogeneities that could cause efficient radiation through a
scattering mechanism.

However, regarding the question of sound generation by turbulence
near surfaces, with supports or edges that extend in a single
direction for distances in excess of an acoustic wavelength, very
little work has yet been attempted. The initial work on these
lines carried out under the present effort has not led to any
significant new contribution. The problem is difficult on two
counts. First the pressure induced by the turbulent flow must
be assessed. In doing this care must be taken that compressi-
bility effects are accounted for in spectral regions where they
are important. Secondly the radiation from sources in the vici-
nity of the edge must be computed taking proper account of the
edge diffraction field. Both these questions seem difficult
ones that require a substantial fundamental study before they
are adequately understood. The pertinence of the problem to
the sound generation properties of turbulent flow near propel-
lor blades or hydrofoil sections makes it an obvious target for
future concentrated effort.

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13. ABSTRACT

This is the final report describing research work on the Theory of Boundary Layer Noise. The main effort has been concentrated on the effects of surface motion. Four published reports describe the details of the effort.

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